Nucleon strangeness form factors and PDFs

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In collaboration with

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χQCD Collaboration

PRD80(09)094503, arXiv:0903.3232
PRD79(09)094502, arXiv:0811.1779

How strange is the nucleon ?

(J.Ellis)

- The (naïve) quark model
  - ➔ Strangeness = Zero ! (too naïve !!?)
- Note that the vacuum is already "strange"

$$\langle \bar{s}s \rangle \simeq 0.8 \times \langle \bar{q}q \rangle$$

- Scalar element
  - y-parameter $y = 2\langle N|\bar{s}s|N\rangle / \langle N|\bar{u}u + \bar{d}d|N\rangle$
  - Pi-N-Sigma term, (neutralino) Dark Matter search, μ -e conversion
- Momentum sumrule
  - 3-5% is carried by strangeness (from experiments)
  - <x>(s) = 0.027(6) ←LatQCD, M.Deka et al. PRD79(2009)094502
- Axial vector

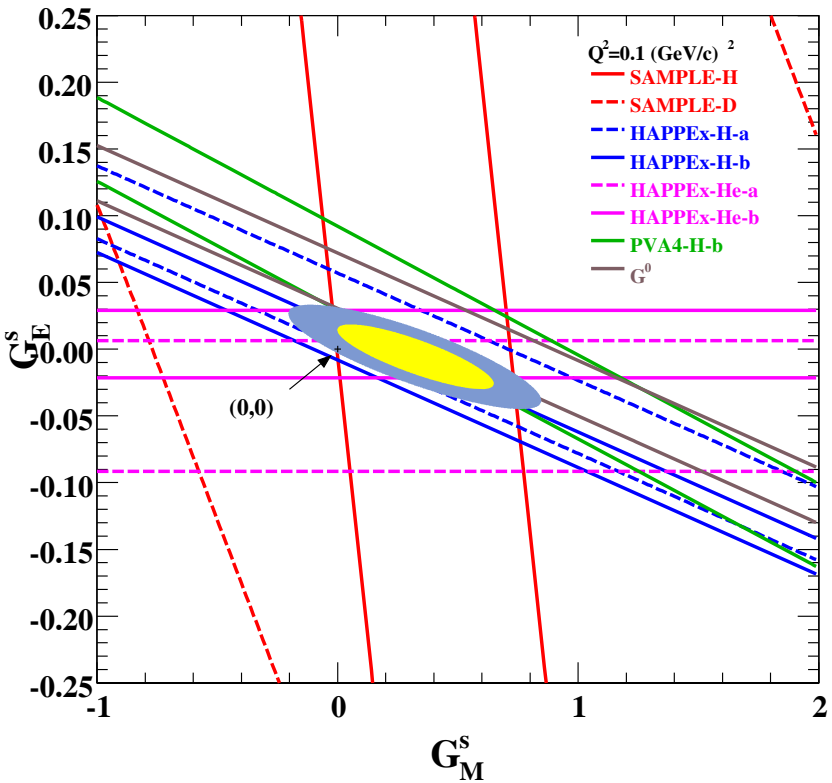
$$\Delta s = -(0.1 - 0) \qquad \Delta s = \int dx [s_\uparrow(x) - s_\downarrow(x) + \bar{s}_\uparrow(x) - \bar{s}_\downarrow(x)]$$

- **How about Vector ?**

Experiments: parity-violating electron scattering (PVES)

- **World averaged data**

$$Q^2 = 0.1(\mathrm{GeV}^2)$$



$$G_E^s(Q^2) = -0.008 \pm 0.016$$
$$G_M^s(Q^2) = +0.29 \pm 0.21$$

J.Liu et al.
PRC76(2007)025202

$$G_E^s(Q^2) = +0.002 \pm 0.018$$
$$G_M^s(Q^2) = -0.01 \pm 0.25$$

Or

$$G_E^s(Q^2) = -0.011 \pm 0.016$$
$$G_M^s(Q^2) = +0.22 \pm 0.20$$

R.Young et al.
PRL99(2007)122003

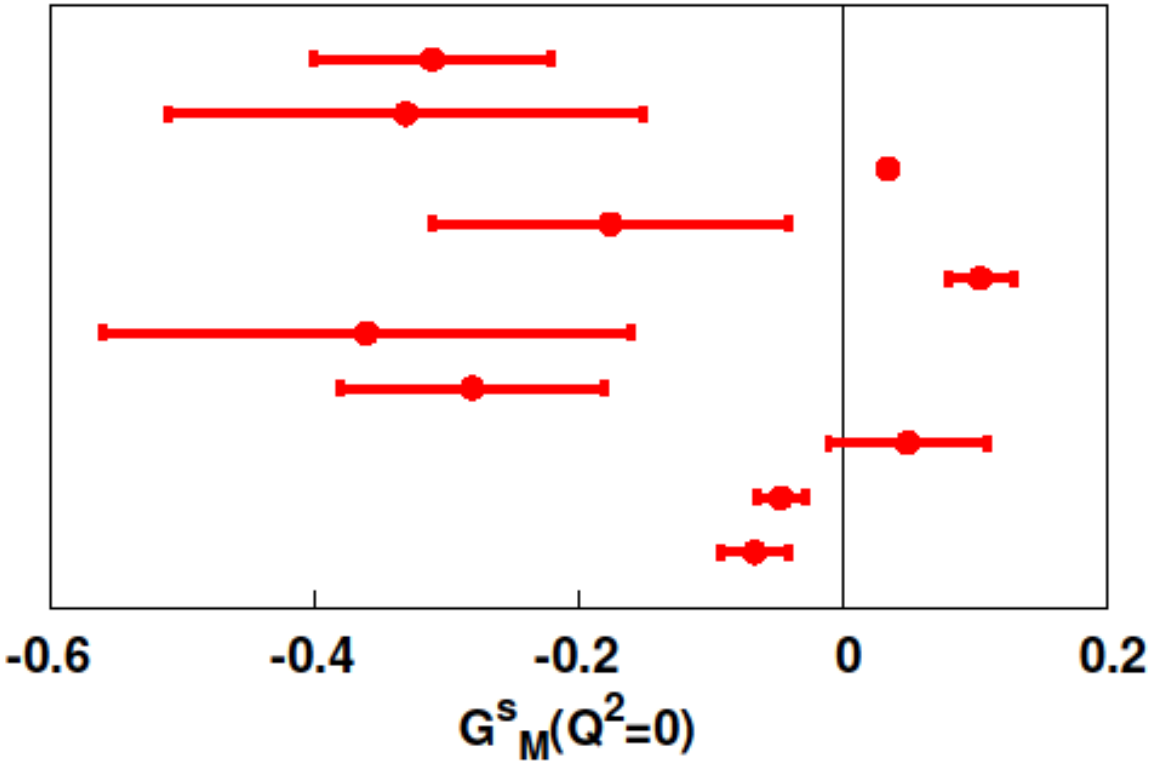
$$\mu_p = +2.79\mu_N$$
$$\mu_n = -1.91\mu_N$$

<~2008

Theoretical status for strangeness form factors

- Theoretical status is quite uncertain

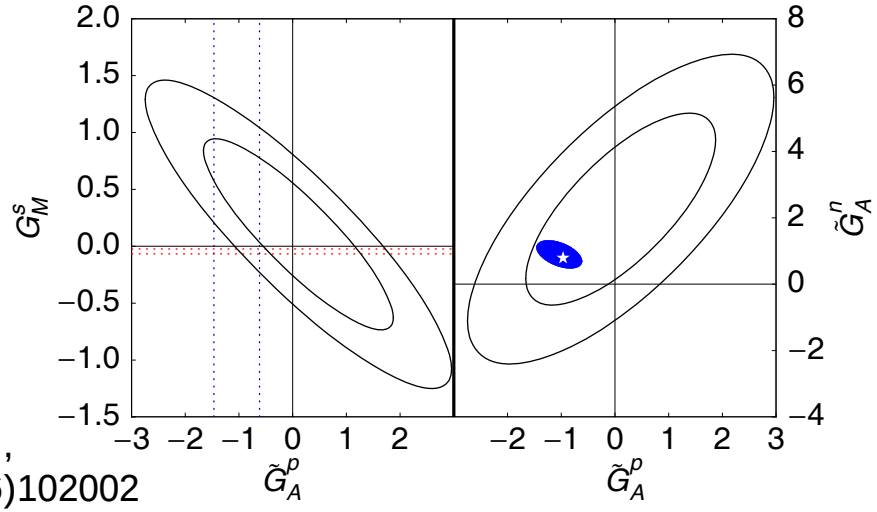
DR w/ pole ansatz
DR w/ scattering kaon clouds
Quark model
Kaon clouds model
Chiral quark-soliton model
Lattice, direct, quenched
Lattice, direct, quenched
Lattice, direct, quenched
Lattice, indirect, quenched
Lattice, indirect, mixed



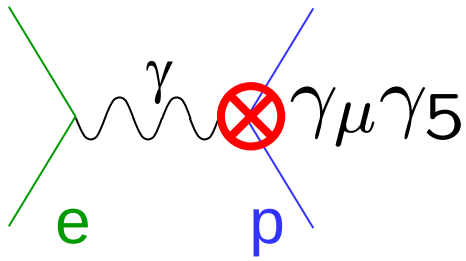
(3rd lat: $Q^2=0.1\text{GeV}^2$)

What is the impact of precise determination of strangeness form factors ?

- **If we can nail them with 3-5 sigmas (both theoretically and experimentally)**
  - Constrain axial form factor G_A^s experimentally
  - Constrain electroweak radiative corrections



Young et al., PRL97(2006)102002



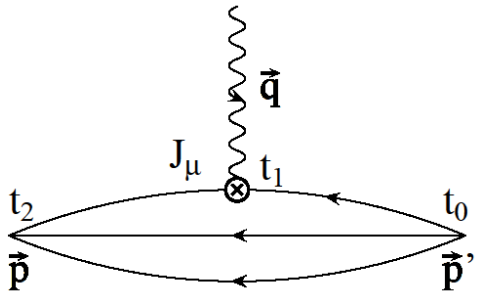
Anapole moment

🡐🡒 Test of EW charge constraint of NP (?)

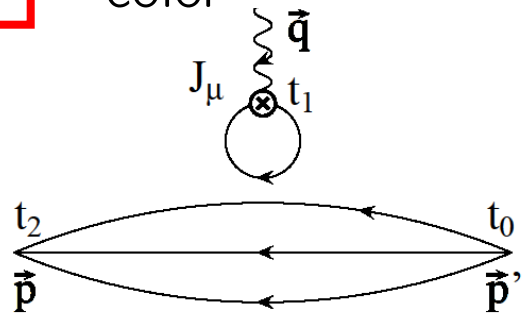
Difficulty in Lattice QCD

- Disconnected Insertion (DI)
  - Inevitable for strangeness calculation, but...
  - **All(source)-to-all(sink)** propagator is necessary
  - Straightforward calculation (has been) **<u>impossible</u>**
    - **O(10^5) inversions** for O(10^6)XO(10^6) matrix

$$\mathrm{Tr}[\Gamma M^{-1}] = \boxed{\sum_x} \mathrm{Tr}^{\mathrm{spin}}_{\mathrm{color}}[\Gamma M^{-1}(x,x)]$$



Connected Insertion (CI)



Disconnected Insertion (DI)

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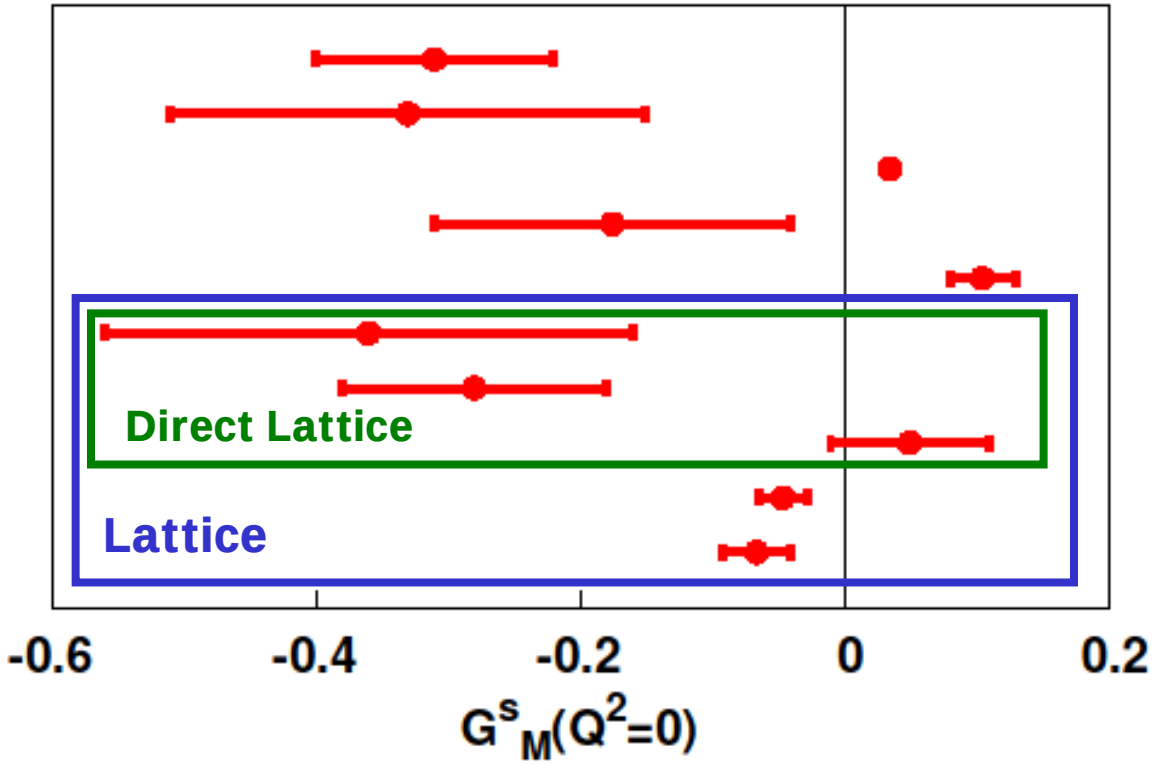


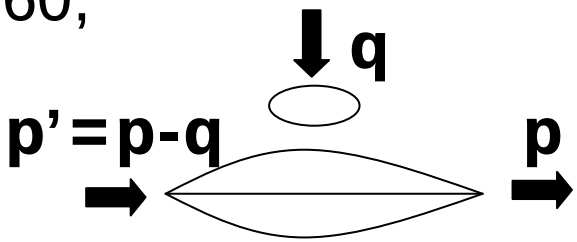
(3rd lat: $Q^2=0.1\text{GeV}^2$)

Strangeness form factors from Lattice QCD with direct calculation of D.I.

Configurations

- Nf=2+1 dynamical clover fermion + RG improved gauge configs (CP-PACS/JLQCD)
  - Beta=1.83, (a^-1=1.62GeV, a=0.12fm)
  - 16^3 X 32 lattice, L=2fm, about 800 configs
  - Kappa(ud)=0.13825, 0.13800, 0.13760, kappa(s) =0.13760
    - M(pi)= 610 – 840 MeV

      T.Ishikawa et al., PRD78(2008)011502



  - Calc 3pt function w/ conserved point-split vector current

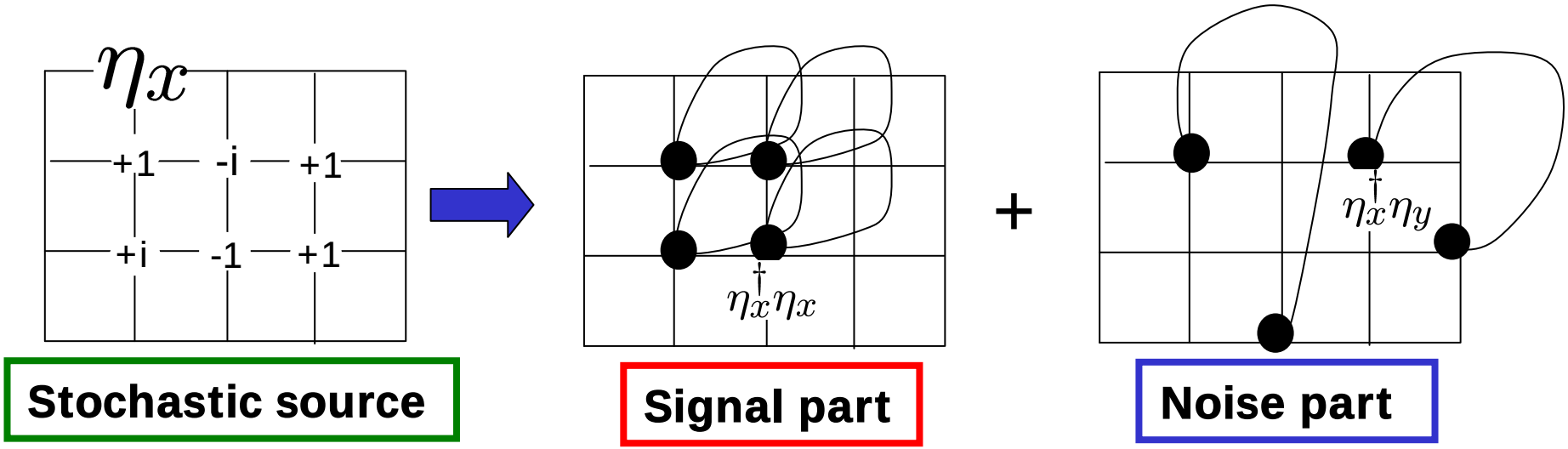
$$\langle p,s|V_\mu(0)|p',s'\rangle = \bar{u}(p,s)\left[\gamma_\mu F_1 - \sigma_{\mu\nu}q_\nu \frac{F_2}{2m}\right] u(p',s')$$

Dirac Pauli

Disconnected Insertion (DI)

- Stochastic Method for DI
  - Noise $$\lim_{L\to\infty}\frac{1}{L}\sum_{l=1}^{L}\eta_i^{l\,\dagger}\eta_j^{l}=\delta_{ij}$$
  - DI loop $$\mathrm{Tr}[\Gamma M^{-1}]=\lim_{L\to\infty}\frac{1}{L}\sum_{l=1}^{L}\eta^{l\,\dagger}(\Gamma M^{-1}\eta^{l})$$

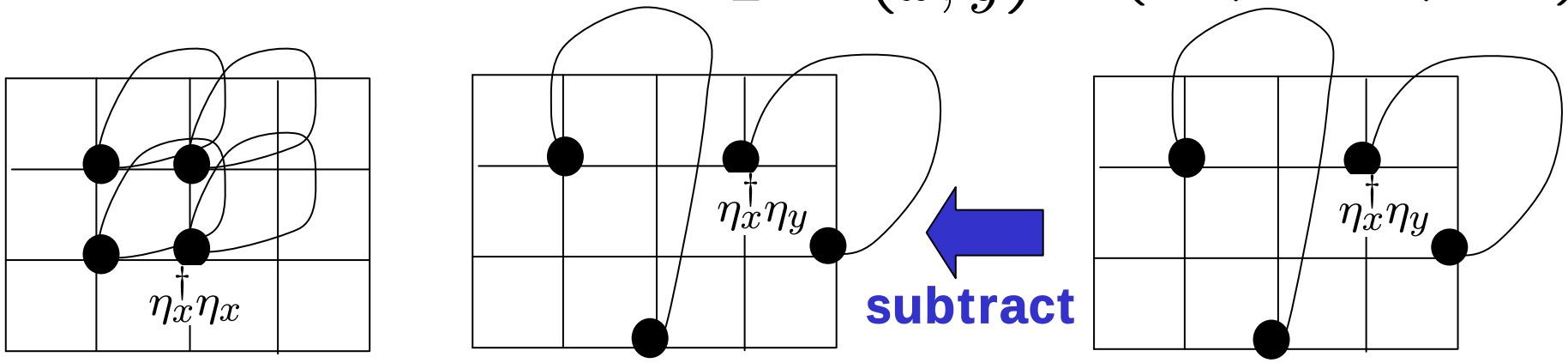
S.-J.Dong, K.-F.Liu, PLB328(1994)130



DI calculation: stochastic method

- The **unbiased subtraction** using **hopping parameter expansion (HPE)** to eliminate off-diagonal noises

$$D^{-1}(x,y) \quad D^{-1} \simeq (1 + \kappa D + \cdots)$$



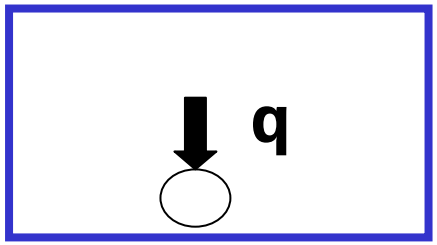
Signal part **Noise part** **Noise cancel part**

➔The error reduces by a factor of 2

Yet, further improvement required

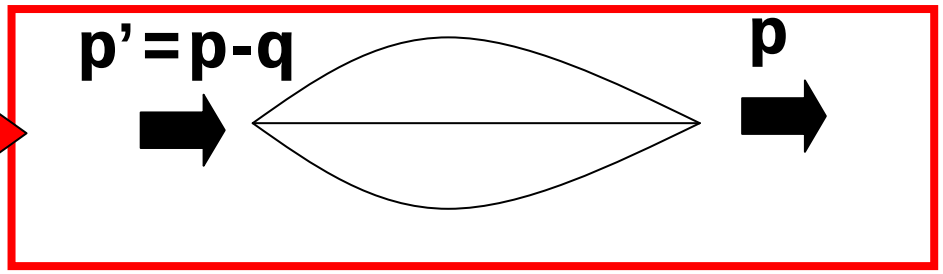
- **Many nucleon sources**

S/N improves by $\sqrt{Nnoise}$



#noise = 600 or 800

S/N improves by $\sqrt{Nsrc}$



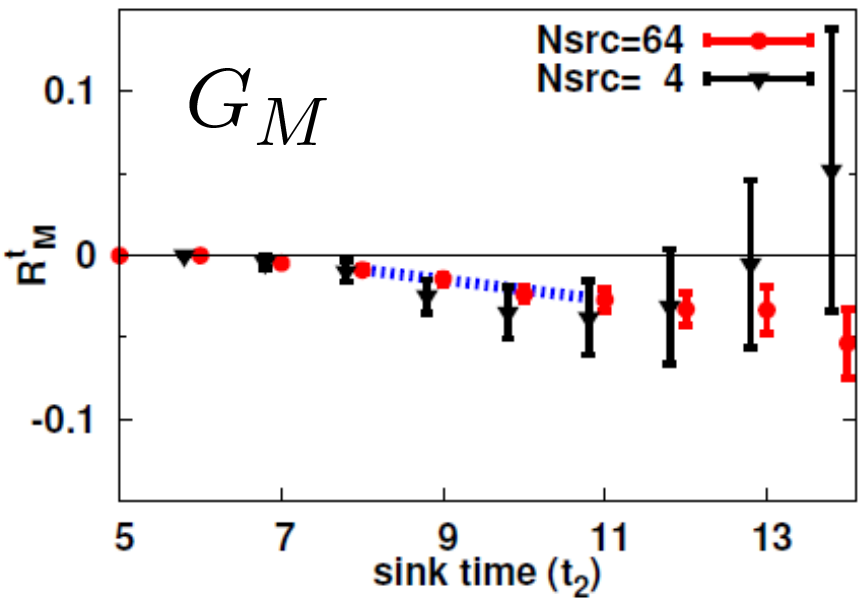
#src = 64 or 82

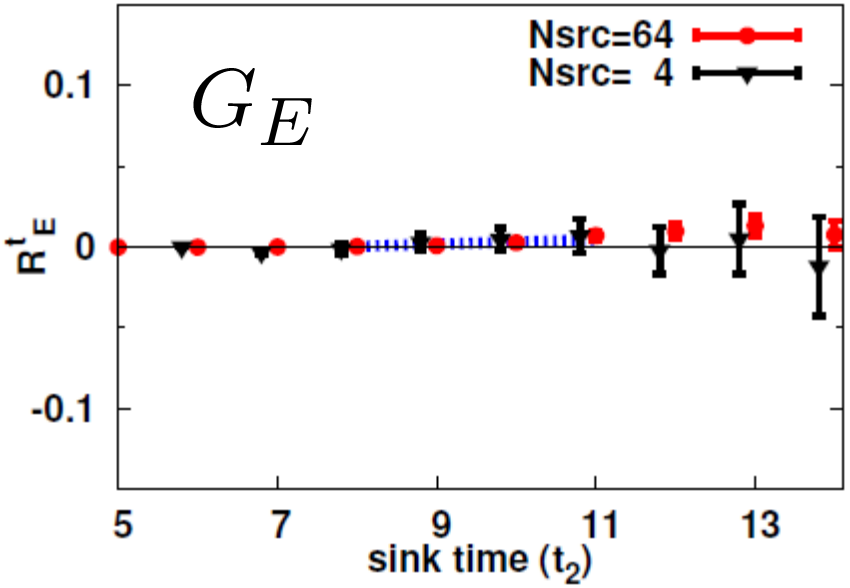
N.B. The calculations of loop part and 2pt part are ***independent*** !

S/N improves almost ideally ! (small auto-corr)

Numerical Results

The result for $G_M(Q^2)$, $G_E(Q^2)$



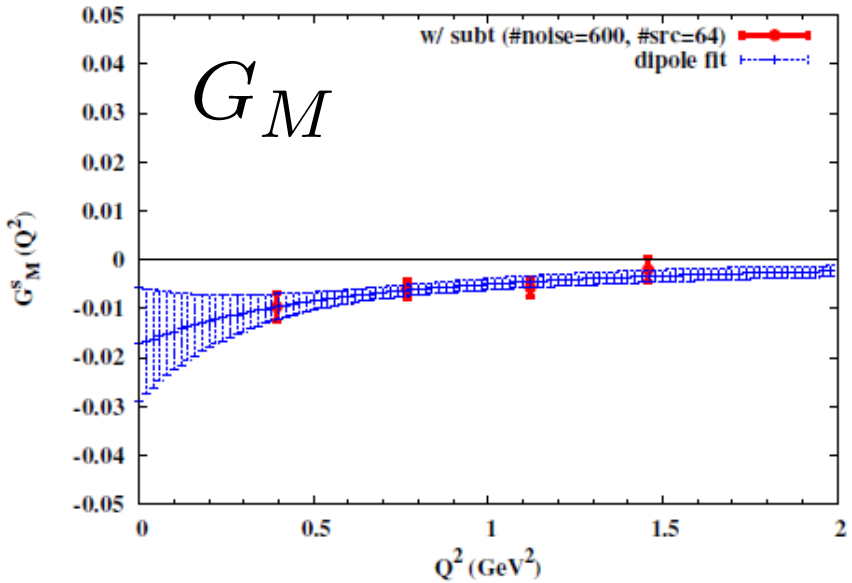


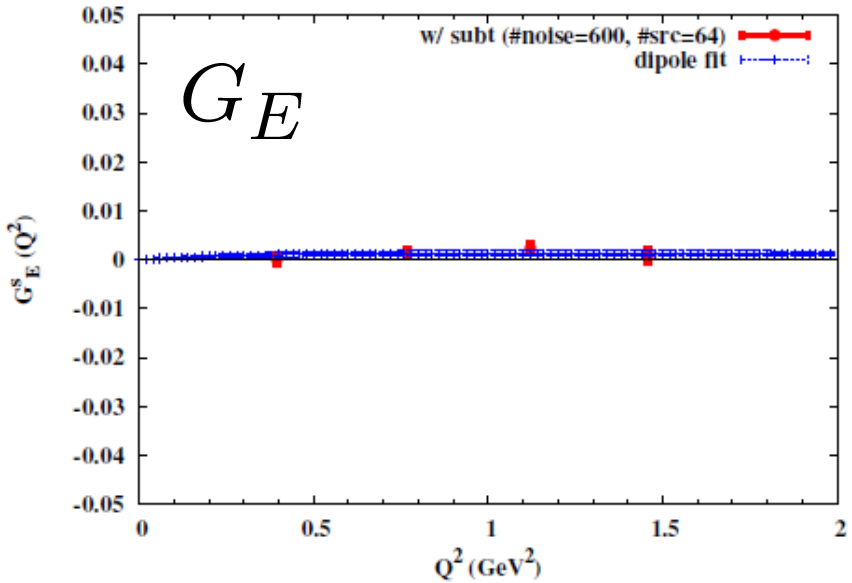
Linear slope corresponds to signal

By increasing the nucleon sources #src=4 ➔ 64, the signal becomes clear

Error bar reduced more than factor 3 !

Q^2 dependence of $G_M(Q^2)$, $G_E(Q^2)$





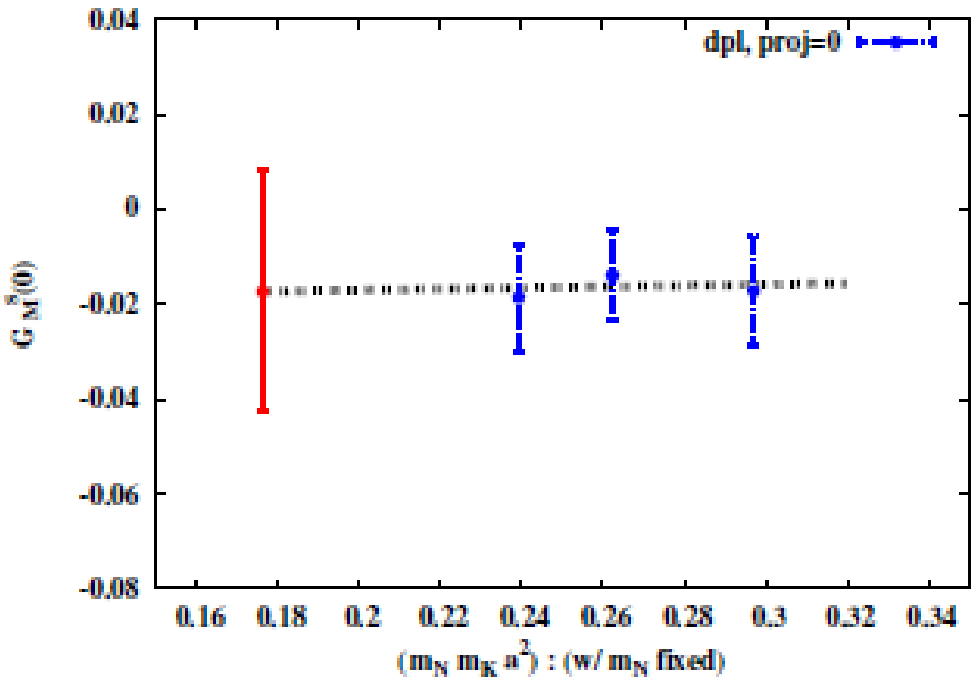
$$G^s_M(Q^2) = G^s_M(0)/(1 + Q^2/\Lambda^2)^2$$

$$G^s_E(Q^2) = g^s_E \cdot Q^2/(1 + Q^2/\Lambda^2)^2$$

🡐 dipole function fit

🡐 cutoff mass Λ is taken from G_M fit (one param fit)

Chiral Extrapolation for $G_M(0)$

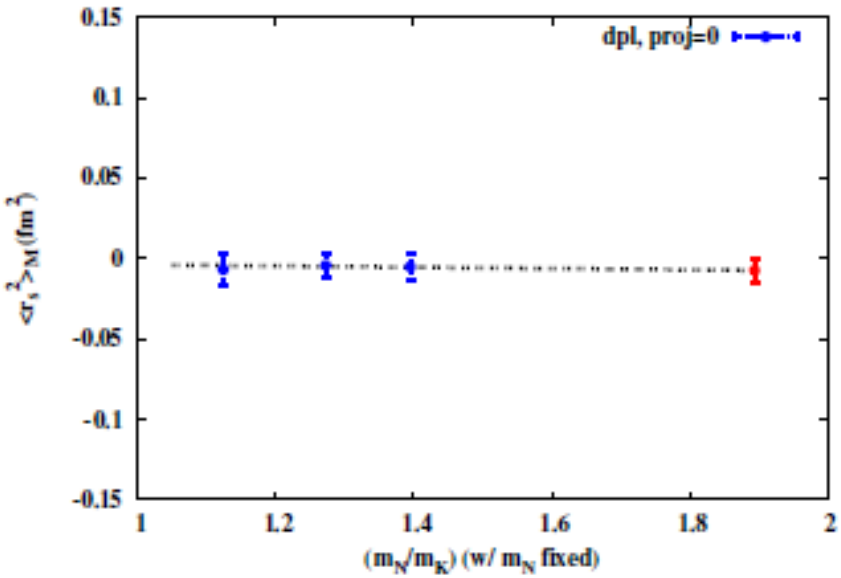


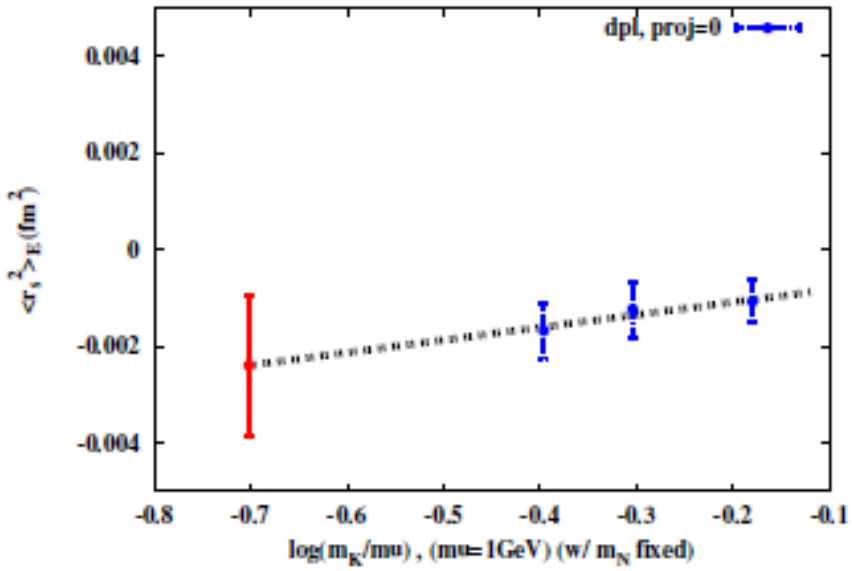
$$G_M^s(0) = -0.017(25)$$

Weak quark mass dependence

The error is about a factor of 10 smaller than previous direct lat calc and/or experiments !

Chiral Extrapolation of $<r^2_M>$, $<r^2_E>$





$$\langle r_s^2 \rangle_M = -7.4(71) \times 10^{-3} (\mathrm{fm}^2)$$

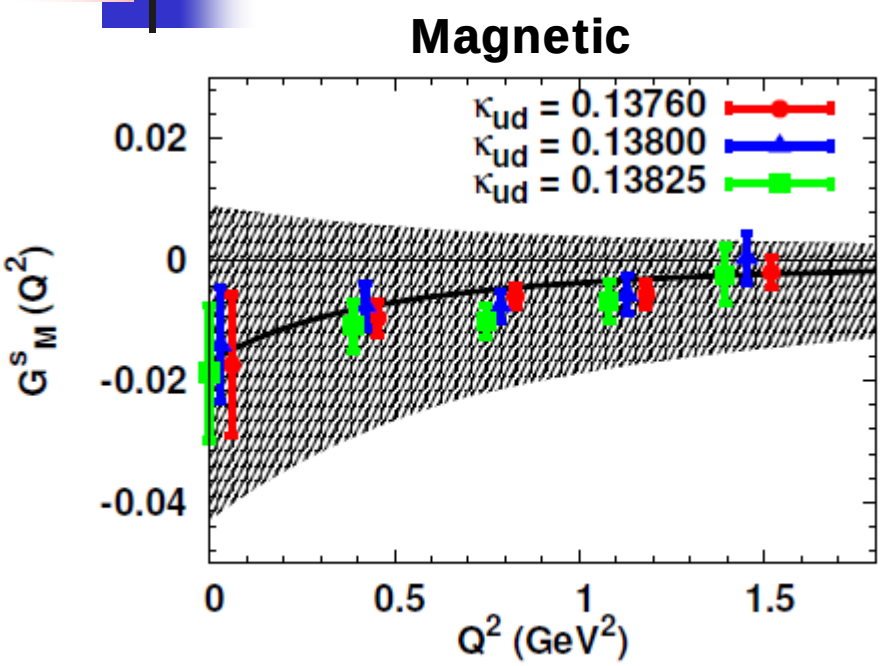
$$\langle r_s^2 \rangle_E = -2.4(15) \times 10^{-3} (\mathrm{fm}^2)$$

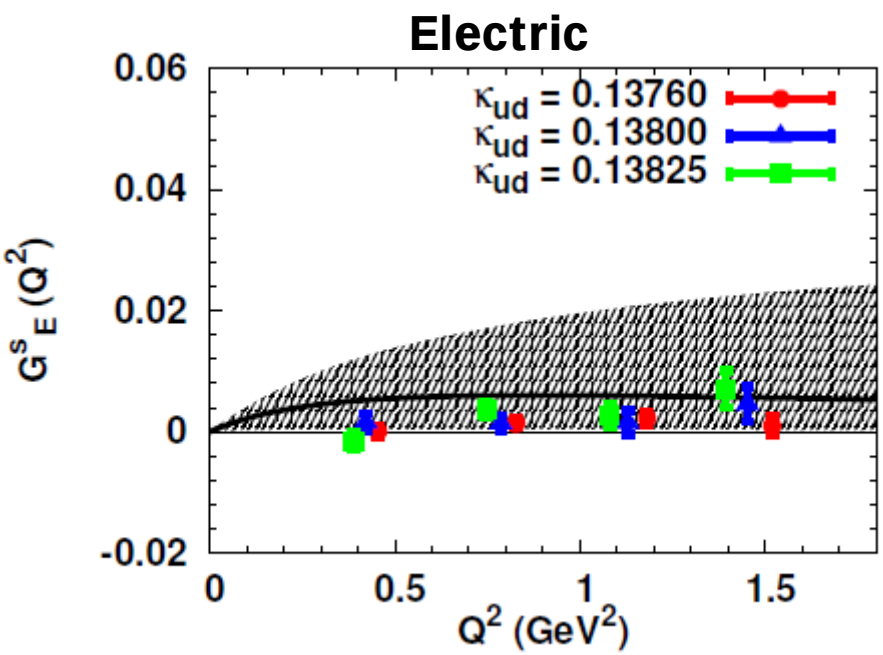
Systematic Uncertainties

- **Uncertainty in Q^2 dependence**
  - We test monopole form instead of dipole form
    - ➔ consistent results, we consider differences as systematic error
- **Uncertainty in chiral extrapolation**
  - We test different extrapolations using nulceon mass mearesd on the lattice
    - ➔ consistent results, we consider differences as systematic error
- **Contamination from excited states** (Roper, S_{11}(N*), etc.)
  - We use different projection to kill S_{11} (1st excited state on lat)
    - ➔ Almost identical results, negligible S_{11} contamination confirmed
- **Finite Volume artifact**
  - Sachs radii are found to be small $|\langle r_s^2 \rangle_{E,M}| \ll 0.1(\mathrm{fm}^2)$
    - ➔ this may indicate small finite V artifact
  - Form factors in isovector (CI) is known to suffer from small finite V artifact
- **Discretization Error**
  - Check on dispersion relation ➔ finite (q a) error is negligible
  - Discretization error in nucleon, kaon mass are around 6-8%

Final Results

The error is about a factor of 10 smaller than previous direct lat calc and/or experiments !





$$G^s_M(0) = -0.017(25)(07)$$

$$\langle r_s^2 \rangle_M = -7(7)(5) \times 10^{-3}(\mathrm{fm}^2) \quad \langle r_s^2 \rangle_E = -2.4(15)(07) \times 10^{-3}(\mathrm{fm}^2)$$

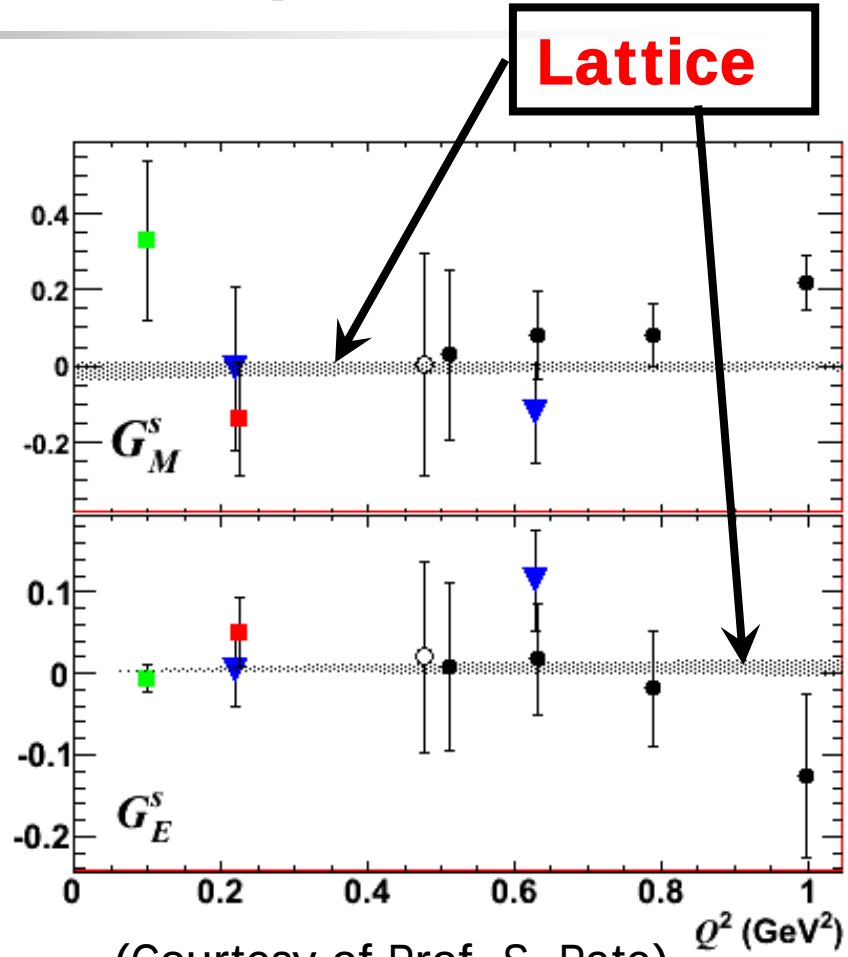
Comparison with experiments

- ● G0 (forward *ep*) + E734 (νp and $\bar{\nu} p$)
- ○ HAPPEx (forward *ep*) + E734 (νp and $\bar{\nu} p$)
  Pate, Papavassiliou & McKee, PRC 78 (2008) 015207

- ■ PVA4 (forward and backward *ep*)
  Baunack et al., PRL 102 (2009) 151803

- ▼ G0 (forward and backward *ep*, and backward *ed*)
  Androic et al., PRL 104 (2010) 012001

- ■ HAPPEx (forward *ep* and e^4He) + G0 (forward *ep*)
  + SAMPLE (backward *ep* and *ed*) + PVA4 (forward *ep*)
  near $Q^2 = 0.1\ \mathrm{GeV}^2$
  Liu, McKeown & Ramsey-Musolf, PRC 76 (2007) 025202



(Courtesy of Prof. S. Pate)

Strangeness PDFs in nucleon

Analysis for <x^2> (D.I.)

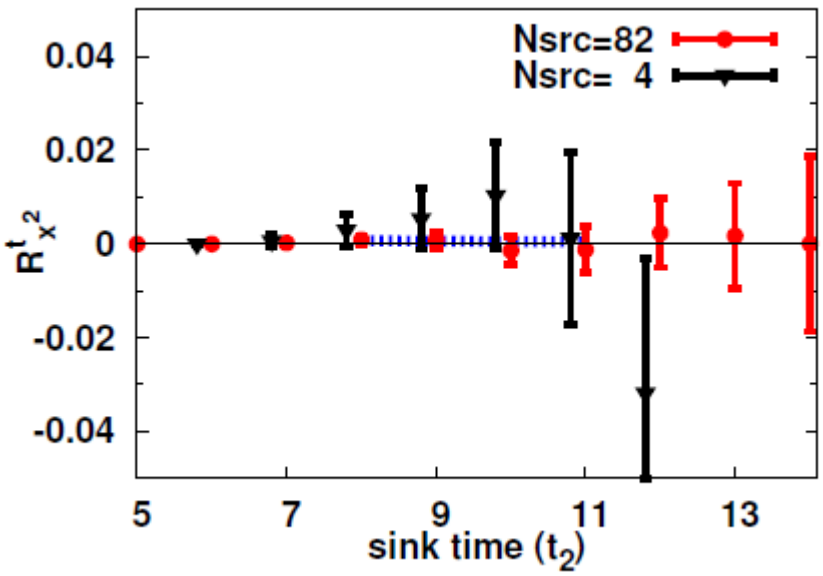
$$\langle x^2 \rangle_{s-\bar{s}} = \int_0^1 dx \; x^2 (s(x) - \bar{s}(x))$$

➔ Information about asymmetry between s and sbar

➔ Could be crucial information to explain NuTeV anomaly (Weinberg angle is 3σ away)

NP or not NP ?

Results for $<x^2>(s)$



(kap=0.13760)

Nf=2+1

Preliminary

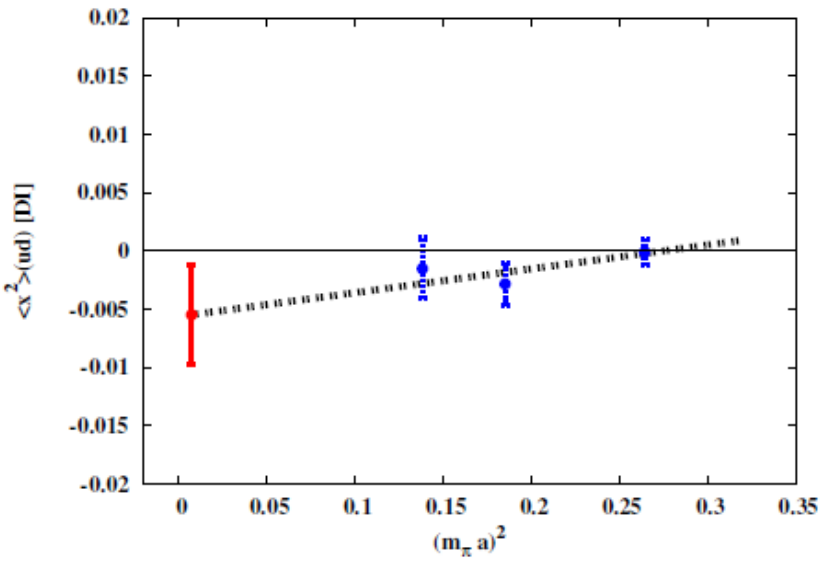
Linear slope corresponds to signal

By increasing the nucleon sources #src=4 ➔ 64, error bar reduced more than factor 3 !

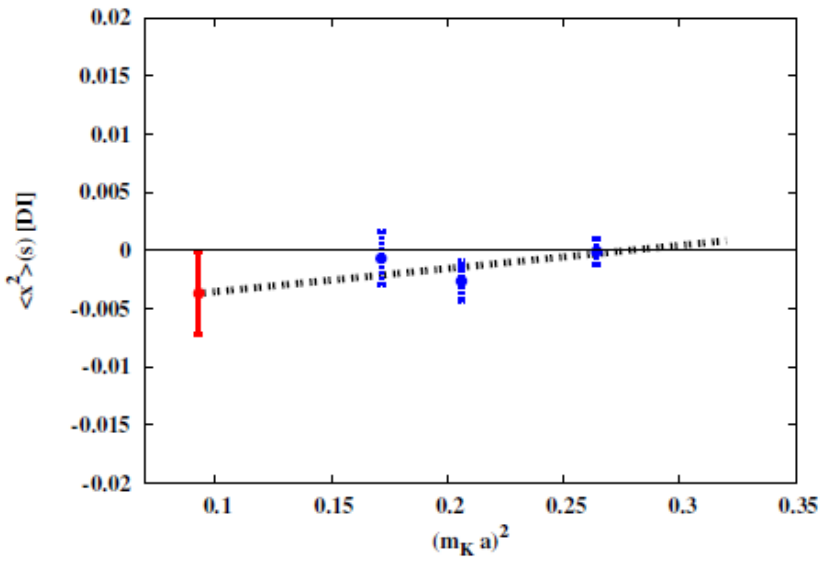
Results for $\langle x^2 \rangle$

Nf=2+1

Preliminary



$\langle x^2 \rangle$(ud) [DI]



$\langle x^2 \rangle$(s)

c.f. $\langle x \rangle_{s-\bar{s}} = 0.0038 \rightarrow$ No NuTeV anomaly

$Z_{\rm pert}(\mu, a) \simeq 1.1$

μ=2GeV

Analysis for <x> (D.I.)

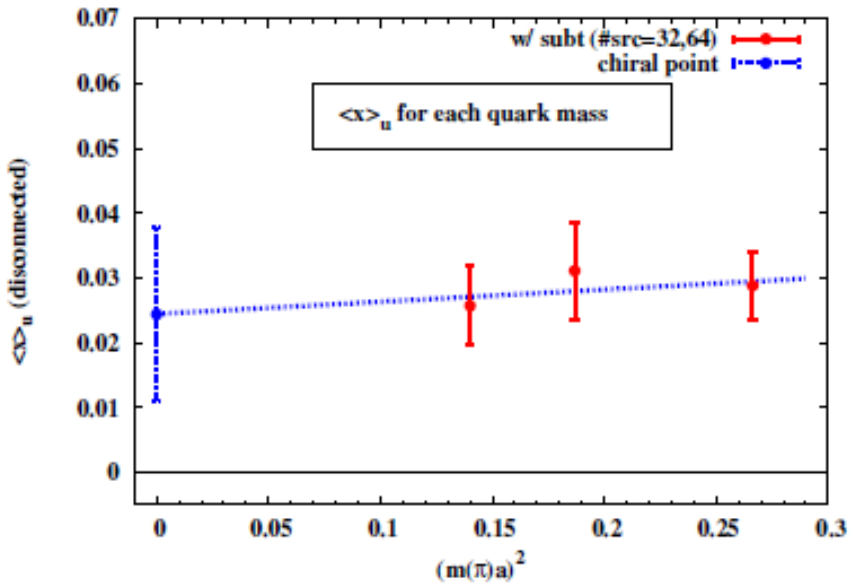
First moment of the nucleon

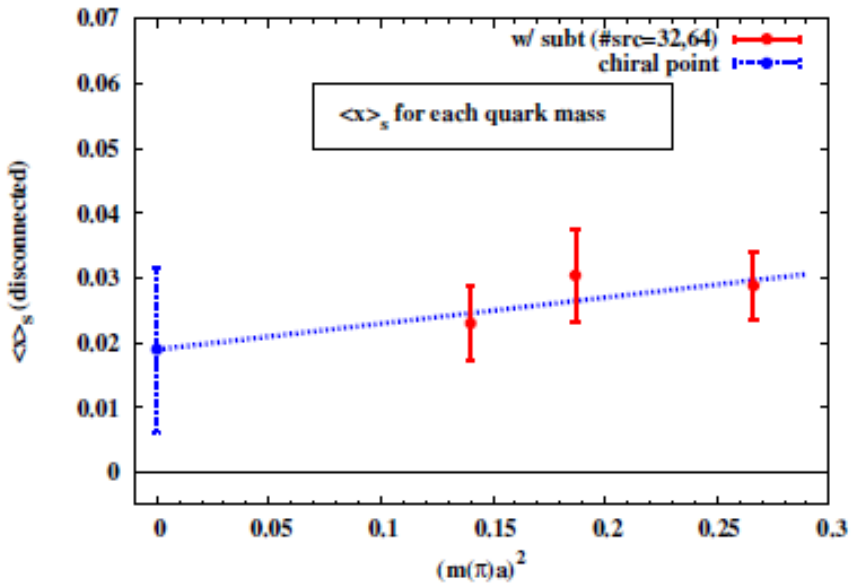
$$\langle x \rangle_q = \int_0^1 dx \; x(q(x) + \bar{q}(x))$$

There have been no calculation for DI !

Chiral Extrapolation for <x>(DI)

Preliminary





<x>(ud) [DI]

<x>(s)

Nf=2+1

From small sampling data:
Full analysis in progress

We expect we can furhter reduce the error by subtraction technique using clover-fermion HPE

Note: The values are not renormalized

Summary/Outlook

- We have studied the strangeness form factors (G_E, G_M) in the nucleon
  - Nf=2+1 clover fermion
  - Disconnected Insertion (DI) has been calculated directly using stochastic method
    - Unbiased subtraction w/ HPE up to 4th order
    - Many nucleon sources are essential to improve S/N

      $G_M^s = -0.015(23), \quad G_E^s = +0.0022(19) \qquad \text{at } Q^2 = 0.1\text{GeV}^2$
    - Analysis for <x>, <x^2> in progress
- Outlook
  - Explicit calc w/ lighter quark mass, larger & finer Lat box
  - Various quantities of D.I., sigma term, nucleon spin, etc.
  - All-to-All using deflation